Inverse problem of super-diffusion on two-dimensional circulant tori

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This work addresses super-diffusive motion on two-dimensional helicoidal tori within a recently proposed inverse stochastic approach that identifies, for any finite substrate with N sites and for any pre-selected $\gamma \gtrsim 1$, a dis- crete time random dynamics such that the mean square displacement of the walkers exactly satisfies $\langle r^2(t) \rangle \sim t^\gamma$ during a finite time interval. Within this framework, super-diffusion is caused by a sequence of time-dependent probability of long distance jumps over the substrate, which is obtained by a combination of stochastic dynamical laws with a Markov Chain (MC) for- malism. In general, its computational complexity increases as a power of N with exponent larger than 2 but, for helicoidal tori, certain properties of circulant matrices allow to obtain analytical expressions requiring much smaller computation times as compared to those necessary when using gene- ral expressions. The results produced by the two MC expressions are in best agreement both with each other and also with the numerical simulations for helicoidal tori up to $N \sim 10^4$. In addition, we show that numerical simula- tions for the usual non-circulant tori also lead to the desired super-diffusive behavior when we use the solutions of the time-dependent probability found for their helicoidal counterpart. The results suggest that the same approach can be further extended to cubic and hyper-cubic lattices.