

# *Practical Information*

FOR

LIFE ASSURANCE AGENTS:

BEING

A SERIES OF TRACTS IN WHICH ACTUARIAL SUBJECTS  
ARE POPULARLY EXPLAINED.

BY

WILLIAM HUGHES, F.I.A.

LONDON :

CHARLES AND EDWIN LAYTON,

56, FARRINGDON STREET, E.C.

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WILLIAM HUGHES,

FELLOW OF THE INSTITUTE OF ACTUARIES OF GREAT BRITAIN AND IRELAND.

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1875.



TO

HENRY HARBEN, Esq.,

TO WHOM HE IS LARGELY INDEBTED FOR HIS PRACTICAL KNOWLEDGE

OF THE BUSINESS OF LIFE ASSURANCE,

THIS LITTLE WORK IS RESPECTFULLY DEDICATED

BY

THE AUTHOR.

LONDON, *July*, 1875.

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## PRACTICAL INFORMATION

FOR LIFE ASSURANCE AGENTS.



### I.

#### PRELIMINARY.

THERE is no want of literature of a certain sort designed for the practical use of Life Assurance Agents. Tracts and leaflets, hints and aids, are numerous enough, and vary very considerably in their merits. For the most part, however, they seem to take but a low view of the intelligence and education of those for whom they are designed. Some of them appear to be framed on the model of religious tracts, others are in the form of pathetic stories, and some that we have seen descend to such doggerel as—

“ When poor Pa died and went to Heaven,  
What grief Mamma endured ;  
But, oh ! her grief was soon relieved,  
For Pa he was insured.”

Surely this sort of thing has not been of much use to agents. But numerous as tracts for agents have been, we think there is room for yet another series.



Every insurance manager or secretary can testify that, although agents as a body are hard-working, intelligent, and business-like men, it is only those who have worked for many years and have accumulated a store of experience, who possess even a moderate amount of knowledge of the elementary principles which underlie the practice of assurance. How much of the time of a manager is occupied in answering inquiries on all sorts of subjects, which ought to be thoroughly understood by an agent; how frequently does he wish that he had at hand some short treatise on assurance matters to which he could refer inquirers, instead of having to write long letters explaining elementary matters!

It is not to be supposed that every agent can hope to acquaint himself thoroughly with actuarial science, but we are quite sure that there are many intelligent and earnest men among them who would take some pains to acquire a little technical knowledge. It is obviously only a very few who possess the time and inclination, or preliminary mathematical knowledge, to study the works of Milne, Baily, Price, and Jones, or the more modern papers of Farr, Chisholm, Tucker, Brown, Woolhouse, Sprague, and other giants of actuarial learning. Between a knowledge of such works as these, and complete ignorance, however, there is a very wide interval, and we are sure that any book or series of papers containing a little elementary information would be cordially welcomed by those agents who desire to obtain an intelligent insight into the principles of life assurance science.

With a view of assisting agents to acquire some small amount of technical knowledge, the present series of papers has been prepared. These papers are, as far as possible, free from mathematical technicalities, and aim at being popular without being unduly super-

ficial. They are designed to give such instruction as will assist agents in their ordinary work, helping them to answer the questions with which they are continually assailed in the prosecution of their business, and aiding them in an understanding of the general theory of life assurance. With this object the following pages contain information on the subjects of Compound Interest, Probabilities, Annuities, Assurances, Mortality Tables, Surrender Values, Bonuses, and kindred subjects. Most of these subjects have been usually considered too abstruse for general comprehension, and so no doubt they are, if comprehension be understood in its widest and literal sense. The study of a lifetime would fail entirely to exhaust even one of these subjects, but we think we can offer a certain amount of information upon them which will interest and instruct, and at the same time be found of practical utility to those for whom they are chiefly intended, namely, life assurance agents.

The objection is often made against popular treatises and epitomes of scientific subjects, that "a little knowledge is a dangerous thing." There is, however, a limitation to the truth of this oft-quoted sentence. It is quite true that there is great danger when a little knowledge is supposed by its owner to be much more extensive than it is. There can, however, be no danger, but on the contrary much profit, in acquiring "a little knowledge," provided it is known to be *only a little*. A dim light is better than utter darkness, and imperfect information than complete ignorance. The intention of these pages is especially to help agents in their work; not in any sense to make superficial actuaries of them. It must be apparent to any agent that he will derive great assistance in his ordinary pursuits if he has some idea, however



imperfect, of the principles on which tables of premiums and annuities have been constructed, and some notion of the general construction and mode of working of the great machine of which he is, so to speak, one of the wheels. Every agent is frequently met by puzzling questions which he cannot answer, and arguments which, though he *feels* they are fallacious, he cannot *show* to be so. Yet these very questions and arguments are generally of a kind that require no learned actuary's knowledge to furnish a reply—a little information is all that is needed, and this little information it is the design of these papers to furnish.

## II.

## INTEREST.

THE first subject which should engage the attention of any person who wishes to obtain an insight into the principles of Life Assurance is that of Interest. The two great foundation stones on which Life Assurance rests are the Statistics of Mortality and the Interest of Money. Hence a clear understanding of the precise nature and effects of Interest is an essential preliminary to the comprehension of the complex problem of Assurance. Simply stated, Interest is the price or consideration paid for the use of money. Capitalists—or, in other words, persons who are in possession of accumulated wealth—are always ready to lend their money, for a certain consideration, on obtaining security for the due repayment of the loan. This consideration is called interest, and the amount of it, or the *rate of interest*, varies generally inversely as the security; that is, the better the security the lower the rate. I do not here propose entering upon the historical part of the subject, or upon any consideration of the causes which regulate the rate of interest; these matters belong to the science of Political Economy, and would be beyond the present purpose.

The nature of Simple Interest is too obvious to need any explanation, and the arithmetical operations



required to compute it are of the simplest character. What is called Compound Interest is much more complex; and although most of the calculations involved in its computation might be performed by the aid of simple arithmetic alone, in the majority of cases such calculation would extend to a most unwieldy length. Hence processes of computation have been invented to shorten the labour, and although they cannot be fully comprehended without some knowledge of mathematics, some notion of their nature may be obtained from the following explanations.

When a lender advances a sum of money, he may either receive the interest at the end of each year, and spend it, or he may re-lend it, either to the original borrower or to some other person, and so obtain *interest upon interest*, or Compound Interest. A little consideration will show that by this latter means the original capital will grow at a constantly increasing rate, and ultimately will become, if the process be continued, many times its original size. Professor De Morgan has remarked that many a man has failed to realise a competence by the unthrifty habit of spending the interest of his investments instead of systematically reinvesting it. It is true that there is some practical difficulty in investing the small interest of a small capital, but this can be done indirectly by paying it to a savings bank, whose business it is to find investments for a large number of small sums, or, better still, by investing it in a Life Assurance. In the latter case the advantage is secured of realising a certain profit independently of the contingency of life. If the assured lives, he is accumulating his savings; if he dies, his representatives are as well off as if he had continued to live to save.

As an illustration of the accumulation secured by

compound interest, let us take the case of £100 invested at 5 per cent., and see how the fund progresses. At the end of the first year, assuming that the interest is receivable yearly, the amount of interest receivable will of course be £5. Now let this be added to the principal, forming a new principal of £105—the interest on this at the end of another year will be £5. 5s.—add this again to the principal, so forming a new principal of £110. 5s. Pursuing the same course, we shall find at the end of the third year the fund will have amounted to £115. 15s. 3d., and in fifteen years the capital will have amounted to more than double its original sum. In 28 years the £100 will have become £411; and if the re-investments were continued for 100 years, the accumulation would amount to £13,150. When very long periods are considered, the amount becomes so astonishing as to appear fabulous. A single Pound accumulated at 5 per cent. for 550 years would amount to more than Four Hundred and Fifty Thousand Millions of Pounds.

The computation of the amount of sums at Compound Interest, may, of course, be effected by pursuing to the required extent the process adopted in the above illustration; but when the term of years is long, it will be found extremely tedious, especially if the rate of interest be also an irregular one, such as  $3\frac{1}{2}$  or  $4\frac{3}{4}$  per cent. In practice, the mode adopted is one which is easily effected by the aid of decimals and logarithms. I would take the present opportunity of strongly recommending those who intend to study the subject treated of in this series of papers, and who are not acquainted with decimals, to devote a short time to their study. Any ordinary treatise on arithmetic contains quite enough information on the subject to enable anyone to follow any



illustration these papers contain, and a few hours, at the most, will suffice in point of time.

If £1 be invested at 5 per cent., the amount at the end of one year will manifestly be £1.05. If this sum be again invested, its amount at the end of another year will be found by ordinary proportion, or rule of three, to be  $1.05 \times 1.05$ , as will appear from the following consideration:—As the principal at the *beginning* of the first year (£1) is to the amount at the *end* of the first year (£1.05), so is the principal at the *beginning* of the second (£1.05) to the amount at the end of the second year ( $£1.05 \times 1.05$ ). A similar calculation will show that, at the end of the third year, the amount will be  $1.05 \times 1.05 \times 1.05$ . In short, for any number of years the amount will be 1.05 multiplied *by itself* as many times as there are years in the term. Similarly, if the rate of interest be, say,  $4\frac{3}{4}$ , then the amount will be 1.0475 multiplied by itself as many times as there are years in the term, and so for any other rate of interest. Now this process, though shorter than that of using pounds, shillings, and pence, and finding the interest for each successive year, will still be found quite tedious enough; but, by the aid of logarithms,\* the product of any number of successive multiplications of any number by itself is readily obtained, and hence the result we are in search of is arrived at without difficulty. Having thus found the amount of £1, we can find the amount of any other principal sum by multiplying the result so obtained by the number of pounds in the original fund.

Hence, we can solve the converse problem—what

\* The art of using logarithms is not difficult of acquirement, and tables of common logarithms may be obtained for a few shillings. The theory on which they are based can only be understood by those who have made some considerable progress in mathematics.

sum now invested will produce a given accumulation at the end of a given number of years? Find what £1 would *amount* to in the given term by the method above given, and by simple proportion we obtain the desired result, by dividing the given accumulation by the amount which £1 would attain to in the given term. If the principal, the amount, and rate of interest be known, the *term* may be found; or if principal, amount, and term be known, we can find the *rate* of interest, but only by using logarithms, there being no direct arithmetical mode of solving these two problems as there is in the case of finding the principal and amount.

Interest is generally payable more than once a year, usually half-yearly, or quarterly. In such case we must divide the rate per cent. by the number of intervals in the year (*i.e.*, by 2 if the interest is payable half-yearly, or by 4 if quarterly), and multiply the number of years by the same. For instance, let the question be:—What will £100 amount to at 4 per cent. in 10 years, interest payable half-yearly? The answer to this will be the same as for the question, what will £100 amount to at *two* per cent. in 20 years? This amount is of course larger than when the interest is receivable and reinvested annually. In the case supposed, the amount of £100 at 4 per cent., in 10 years, *yearly* interest, is £148.024, but with *half-yearly* interest the amount is £148.594. For short terms the difference is inconsiderable, but becomes of great importance when the period is long.

Closely allied to Interest is Discount. This is often supposed to be the same thing under another name, the practice which actually prevails having obscured the real difference which there is between them. Many commercial men on being asked for a definition



of the terms "interest" and "discount," would give an erroneous reply:—they would assert that interest is the price paid for money lent, payable with the repayment of the loan, and that discount is a deduction or allowance made for the immediate payment of money not yet due. So far, the answer is correct. But put an actual case. What is the *interest* on a loan of £100 at 5 per cent. at the end of a year? The answer would be £5; and this is true. But what is the *discount* for immediate payment of £100 due a year hence, at 5 per cent.? Most men would reply £5, but this is *not* true. The true discount is £4. 15s. 2½d. A little consideration will show that this is the case. Suppose a person has engaged to pay me £100 twelve months hence, but is desirous of paying at once, assuming money to be worth 5 per cent., what should I allow him? Evidently such a deduction that if I re-invest what he pays me at 5 per cent., I should be, at the end of the year, in exactly the same position as if he had carried out his original contract. Now if I allow him £5, as it would be generally supposed I ought, he would, of course, pay me £95, and if I invest this at 5 per cent. I shall at the year's end receive, not £100, but only £99. 15s., thus being the loser of 5s. But if I allow him only £4. 15s. 3d., the *true* discount, he will pay me £95. 4s. 9d., and if I invest this amount at 5 per cent., I shall receive at the year's end the principal, £95. 4s. 9d., and the interest upon it, £4. 15s. 3d., together £100, and so be in exactly the same position as if my debtor had fulfilled his original engagement.

## III.

## ANNUITIES CERTAIN.

THE word "Annuity" is derived from the word *annus*, a year, and in its simplest signification, means a payment of money *yearly*. In practice an annuity is often payable at half-yearly, quarterly, or other periodical intervals; but calculations are generally based on the assumption that annual payments are intended. Annuities are of various kinds, such as annuities certain, life-annuities, temporary, reversionary, intercepted, contingent, and others. The varieties of life-annuities will be considered in their proper place; at present we will take the simple case of an annuity *certain*. By this is meant a sum of money paid annually for a definite period, the payments of which do not depend upon the duration of the life of any person, or on any other uncertain event. A familiar example of such an annuity is the rent payable under an ordinary *lease*. This rent is payable to the land-lord, or (in the event of his death during the term of lease) to his representatives, by the lessee or his representatives. It is not dependent upon the life of either the grantor or of the person to whom it is granted, and being thus independent of contingencies, may properly be called an annuity *certain*.

By the *present value* of an annuity, is meant the amount which it is necessary to have in hand to pro-



vide for the due payment of the annuity from time to time, as it becomes due; and it is evident that this depends upon two things—(1) the term, or number of years, for which the annuity is granted, and (2) the rate of interest at which the money is invested. If money made no interest, of course the value would be exactly found by multiplying the annual payment by the number of years. For example, the sum required to provide for an annuity of £10 for ten years would be exactly £100. In such a case the £100 would be put by, £10 taken out at the end of each year, and at the end of the term the capital would, of course, be exactly exhausted. In practice, however, compound interest is always assumed. The capital, or present value, is supposed to be invested at a given rate of interest, and the higher this rate, the smaller evidently is the amount of capital required. To compute the value of an annuity by the ordinary methods of arithmetic, especially when the term is long, is a very tedious operation. We have seen in our Tract on Interest that we can readily arrive at the value of a sum payable any number of years hence, or, in other words, what amount now invested will amount to the desired sum in a given time. Now, if we want to arrive at the value of an *annuity* or annual payment of say £1, we have only to find the present value of the several payments separately, and then add the results together. For example, suppose it is desired to know what sum is necessary to provide for the annual payment of £1 for six years at 5 per cent., we might proceed thus:—Find the present value of £1 payable six years hence, five years hence, four years hence, and so on, and then add the results together—evidently a tedious operation, and if the term were much longer, the calculation would involve an in-

tolerable amount of labour. Mathematical science, however, provides us with a short method of arriving at the same result; and, without attempting to demonstrate it, we may simply state that the value of an annuity certain of £1 for any term of years may be found by taking the value of the *last* payment only, subtracting this value from £1, and dividing the result by the rate of interest for £1. The value of the last payment, expressed in decimals, is, in the case supposed, .74622. Subtract this from 1, and the result is .25378. Divide this by .05, the interest on £1 for one year, and we obtain 5.0756, which is the present value of the annuity of £1 for six years. If the annuity be for, say, £10, we multiply the result by 10, and similarly for any other sum. We can readily test the sufficiency of the value so obtained by ordinary arithmetic, as follows:—In the example we have found that £5.0756 is the value of the annuity of £1 for the given sum at 5 per cent. If the annuity be £10, the value is £50.756, or £50. 15s. 1 $\frac{3}{4}$ d.; that is, if we invest £50. 15s. 1 $\frac{3}{4}$ d. at 5 per cent., we shall be able to make six annual payments of £10. That this is the case the following will show.

|                                    | £   | s.  | d.  | £               | s. | d.               |
|------------------------------------|-----|-----|-----|-----------------|----|------------------|
| The Fund at commencement is        | ... | ... | ... | 50              | 15 | 1 $\frac{3}{4}$  |
| Add Interest at end of first year  | ... | 2   | 10  | 9               |    |                  |
| Deduct Annuity                     | ... | 10  | 0   | 0               | 7  | 9                |
|                                    |     |     |     |                 |    | 3                |
| Fund at end of first year          | ... | ... | ... | 43              | 5  | 10 $\frac{3}{4}$ |
| Add Interest at end of second year | ... | 2   | 3   | 3 $\frac{1}{2}$ |    |                  |
| Deduct Annuity                     | ... | 10  | 0   | 0               | 7  | 16               |
|                                    |     |     |     |                 |    | 8 $\frac{1}{2}$  |
| Fund at end of second year         | ... | ... | ... | 35              | 9  | 2 $\frac{1}{4}$  |
| Add Interest at end of third year  | ... | 1   | 15  | 5 $\frac{1}{2}$ |    |                  |
| Deduct Annuity                     | ... | 10  | 0   | 0               | 8  | 4                |
|                                    |     |     |     |                 |    | 6 $\frac{1}{2}$  |
|                                    |     |     |     |                 | 27 | 4                |
|                                    |     |     |     |                 |    | 7 $\frac{3}{4}$  |



|                                                            | £  | s. | d.               | £ | s. | d.              |
|------------------------------------------------------------|----|----|------------------|---|----|-----------------|
| Fund at end of third year ... ..                           | 27 | 4  | 7 $\frac{3}{4}$  |   |    |                 |
| Add Interest at end of fourth year ... 1 7 2 $\frac{1}{4}$ |    |    |                  |   |    |                 |
| Deduct Annuity ... ..                                      | 10 | 0  | 0                | 8 | 12 | 9 $\frac{1}{4}$ |
| Fund at end of fourth year ... ..                          | 18 | 11 | 10 $\frac{1}{2}$ |   |    |                 |
| Add Interest at end of fifth year ... 0 18 7               |    |    |                  |   |    |                 |
| Deduct Annuity ... ..                                      | 10 | 0  | 0                | 9 | 1  | 5               |
| Fund at end of fifth year ... ..                           | 9  | 10 | 5 $\frac{1}{2}$  |   |    |                 |
| Add Interest at end of sixth year ... 0 9 6 $\frac{1}{2}$  |    |    |                  |   |    |                 |
| Deduct Annuity ... ..                                      | 10 | 0  | 0                | 9 | 10 | 5 $\frac{1}{2}$ |
| Fund is exactly exhausted at the end of the sixth year     | 0  | 0  | 0                |   |    |                 |

What is technically called the "value" of an annuity, is, in popular language, called the "number of years' purchase." Thus, in the example just given, the annuity for six years, at 5 per cent., is said to be worth rather more than *five years' purchase*.

An annuity payable *for ever* is termed a "perpetuity." At first sight it might be supposed that no sum of money, however large, would be sufficient to provide for so formidable an engagement. This would actually be the case if money produced no interest, for of course the fund would have to be infinite if an infinite number of annual payments are to be made out of it: When, however, the fund is invested at a certain rate of interest, if that interest only is paid away in the form of an annuity, the number of possible payments is illimitable. For instance, £100 invested at 5 per cent. produces £5 a year for ever, the capital remaining untouched, so that a perpetual annuity of £1 may be granted for a consideration of £20. If, however, any one were to undertake to grant an annuity of £1 for a consideration of £19, he would find that, if he invested the purchase-money at 5 per cent., the capital would be exhausted in 62 years.

If the first payment of an annuity is to be made one year hence, and thenceforth to be continued annually, it is said to be an *immediate annuity*. When the annuity is not to be entered upon until the expiration of a certain number of years, it is said to be a *deferred annuity*. This is sometimes called a *reversionary annuity*, but the former term is the one usually adopted when reference is made to annuities certain, the latter being generally used in connection with life annuities.

In books of Tables of Annuities will usually be found a table of the *amount of annuities forborne*. These tables show the sum to which an annuity will amount if neither interest nor capital be touched, and strikingly exemplify the wonderful power of compound interest. An example will best illustrate the meaning of these Tables. Suppose a youth of 16 years of age determines to put by £10 a year, and to invest it at 5 per cent. interest, and to add the interest to the capital at the end of every year, instead of drawing it and spending it. If this course be adopted until he reaches the age of 70, the fund will have amounted to no less than £2,587—a striking illustration of the value of small savings carefully managed.



## IV.

## PROBABILITIES.

THE science of Life Assurance is based mainly upon a consideration of the Probabilities of life. The uncertainty of life is a matter of daily observation, and it is impossible to foretell how long any given person may live. We shall see, however, when we come to the subject of Tables of Mortality, that if we take into consideration a sufficiently large number of lives, it is possible to predict with a great degree of accuracy the number of deaths which will take place in each year, until all the lives under observation have ceased to exist. The application of the Theory of Probabilities to the practical business of Assurance will be considered hereafter. For the present we shall confine ourselves to a few brief remarks on Probabilities or Chances generally.

Much scepticism exists in many quarters as to the nature and trustworthiness of this theory. The reason of this scepticism is to be found where the origin of nearly all scepticism resides—namely, in ignorance and prejudice. It is often supposed that the Theory of Chances is based upon a notion subversive of a belief in the Divine existence and providence. A true conception, however, of what we call chance will be found to be perfectly consistent with a belief in the Divine government and foreknowledge. The fallacies which

underlie all the prejudices against the use of the word Chance consists in a wrong notion of its true meaning. Chance is not a property of events, as colour and form are properties of things, but is merely the indication of the state of our minds with regard to them. This will, perhaps, be made clearer by a simple illustration. Suppose I bring into a room a box containing a sovereign and a shilling. In my absence one of the coins is taken out—say the sovereign—and is shown to every person in the room. Here is a determined event, fixed and known—there is no chance about it. On my return I am informed that one of the coins has been taken out and is to be given to me, but I am not told which of the coins has been drawn. Now, what is the state of my mind with regard to this transaction? In my mind there exists a notion of *Chance*, and it evidently makes no difference, so far as this notion is concerned, whether the person who drew the coin resolved beforehand to take the sovereign, or took the first coin that came to hand, so long as I am ignorant of the fact. For myself, all I can say is that it is equally likely that I shall receive one shilling or twenty; in other words, that it is an even *Chance* whether I get the one or the other. It is not, perhaps, quite so evident that my expectation is exactly worth 10s. 6d., but it is nevertheless true, and any person who had exactly the same knowledge of the facts of the case as myself, would equitably purchase my right to the drawn coin for this amount.

There are very few future events of which we may say we are certain they will occur—not many of which we may say that they are so probable that we feel all but certain they will take place. There are other events of which we say they are quite possible, but not probable; and others again of which we should



say they are in the highest degree unlikely to happen. Now, here we have a sort of descending scale of probabilities, and in each set of our supposed scale we can place a variety of events. For example:—That the sun will rise to-morrow we may assert with confidence—it is practically a certainty. That a given young person now alive and well will be alive to-morrow is highly probable, but not certain—that he will be alive two years hence is also probable, but obviously much less so than that he will be alive to-morrow—far less probable is it, though quite possible, that he will live to attain the age of 80; and lastly it is very improbable that he will attain the age of 100. Every one will admit the force of these general remarks, but few who have not given special attention to the subject are able to give any clear idea of the comparative force or measure of these respective degrees of likelihood. The Science of Probabilities enables us, however, to give a very accurate estimate, and to present this estimate in an arithmetical form.

In order to measure anything—whether it be a material substance, such as cloth or wood, or a physical condition, such as heat or cold—we must have a *standard* of comparison, a yard or inch, or a degree of the thermometer. Similarly, in measuring probabilities, we must have a standard, and the most convenient standard for this purpose is *absolute certainty*. Every unknown event produces on our minds, so far as the probability of its occurrence is concerned, an impression which bears some relation to certainty—it is either certain, nearly certain, or more or less remotely certain. Now, to measure the degrees of distance from complete certainty with accuracy, and to give them a convenient form of expression, we represent absolute certainty by the figure *one*, or unity, and hence the degrees of

probability can be represented by a fraction. To illustrate this, suppose a bag which we know contains a number of black balls, and only black balls. If one is to be drawn, we are of course certain that it will be black. Let this *certainty* be denoted by 1. Now, suppose a bag containing one white and one black ball—what is the probability that if I draw one ball at random it will be black? As there are but two balls, and I must draw one of them, it is sure to be either white or black, but we can assign no reason why it should be one rather than the other; the probability of drawing white is exactly equal to the probability of drawing black. It will be seen, therefore, that the chance of drawing a white ball will be properly represented by  $\frac{1}{2}$ . In popular language, we say it is an “even chance.” If there be three balls of different colour—say white, black, and red—the probability of drawing a given colour, as white, will be  $\frac{1}{3}$ . It is 2 to 1 against it. That is, out of the three *possible* events, two are other than the required result—the third is the one we want, and all are *equally* possible. When we come to more complicated cases, the mathematical probability, as it is called, is not so easily arrived at, nor can it be so immediately perceived to be true. Suppose, for instance, the bag contained four white and six black balls—five are to be drawn, and it is required to know what is the chance that two, and *two only*, shall be white. Now, it can be shown that there are 252 sets of 5 balls each which can be drawn out of 10, and of these 120 sets will, in the case supposed, consist of two white and three black balls. So that the probability of drawing 2 white and 3 black out of 4 white and 6 black will be  $\frac{120}{252}$ , or  $\frac{10}{21}$ .

The application of the Theory of Chances to Life Assurance will be more obvious as we proceed. Mean-



while, it will be enough to give one illustration in this place of a very simple case. It has been observed, as an actual fact, that out of 12,000 persons aged 20, about 80 die before attaining the age of 21. Now, if we take any one of these 12,000 who is at present in ordinary health, it is impossible to say with certainty whether he will be one of the 80 who will die, or of the 11,920 who will survive—but we feel that the probability is in favour of the latter event, and in fact the *measure* of that probability is exactly  $\frac{11,920}{12,000}$ , or, what is the same thing, the chance of his dying is  $\frac{80}{12,000}$ . We shall, however, reserve any further illustration of this subject until we have considered the nature of Mortality Tables, which will form the subject of our next Tract.

## V.

## TABLES OF MORTALITY.

NEARLY every science has originated in the observation of facts. Certain phenomena have been observed, and the circumstances attendant on their occurrence noted—thence inferences have been drawn, and frequently some general law has been discovered which satisfactorily accounts for the facts observed. This process being completed by the philosopher, the practical man takes up his part, and working as it were outwards from the central truth, and building upwards from the foundation which the philosopher has laid, constructs a fabric adapted to the ordinary wants and requirements of society. It has been thus with Life Assurance. Philosophical observers have turned their attention to the statistics of the duration of human life, and presented the result of their observations in an orderly form. Mathematicians have analysed these results, and subjected them to various tests and trials, presenting a new set of facts, or, perhaps more strictly, the same facts in combination with others and in new forms, ready for the immediate use of business men, who, in their turn, have devised and carried out that system of payments and benefits dependent on the duration of life which we know as Life Assurance.



A Table of Mortality is a concise exhibition of the results of observations on the duration of life. The most simple way of forming such a table would be to select a large number of infants born alive, all of the same sex and social station, and to note from year to year the number who died in each year till all were extinct. A column of ages, accompanied by a column of the number of deaths at each age, with another showing the number of survivors at the end of each year, would be a Table of Mortality in its most usual form. It might take 100 years to complete such a table, and, in practice, other methods have been adopted, the details of which it is impossible here to particularise, based upon careful observations of the population, births, and deaths, in certain places or societies, extending over a limited number of years. Assuming that the same causes of mortality have been at work during the whole duration of the lives observed, as were efficient during the time of observation, the result of such a process would evidently be the same as if the longer method above spoken of had been adopted. The following is a specimen of a mortality table as usually presented, being the celebrated Carlisle Table :—

| Age.     | Living.      | Dying.      | Age.     | Living.     | Dying.   |
|----------|--------------|-------------|----------|-------------|----------|
| 0 .....  | 10,000 ..... | 1,539 ..... | 13 ..... | 6,368 ..... | 33 ..... |
| 1 .....  | 8,461 .....  | 682 .....   | 14 ..... | 6,335 ..... | 35 ..... |
| 2 .....  | 7,779 .....  | 505 .....   | 15 ..... | 6,300 ..... | 39 ..... |
| 3 .....  | 7,274 .....  | 276 .....   | 16 ..... | 6,261 ..... | 42 ..... |
| 4 .....  | 6,998 .....  | 201 .....   | 17 ..... | 6,219 ..... | 43 ..... |
| 5 .....  | 6,797 .....  | 121 .....   | 18 ..... | 6,176 ..... | 43 ..... |
| 6 .....  | 6,676 .....  | 82 .....    | 19 ..... | 6,133 ..... | 43 ..... |
| 7 .....  | 6,594 .....  | 58 .....    | 20 ..... | 6,090 ..... | 43 ..... |
| 8 .....  | 6,536 .....  | 43 .....    | 21 ..... | 6,047 ..... | 42 ..... |
| 9 .....  | 6,493 .....  | 33 .....    | 22 ..... | 6,005 ..... | 42 ..... |
| 10 ..... | 6,460 .....  | 29 .....    | 23 ..... | 5,963 ..... | 42 ..... |
| 11 ..... | 6,431 .....  | 31 .....    | 24 ..... | 5,921 ..... | 42 ..... |
| 12 ..... | 6,400 .....  | 32 .....    | 25 ..... | 5,879 ..... | 43 ..... |

| Age.     | Living.     | Dying.    | Age.      | Living.     | Dying.    |
|----------|-------------|-----------|-----------|-------------|-----------|
| 26 ..... | 5,836 ..... | 43 .....  | 66 .....  | 2,894 ..... | 123 ..... |
| 27 ..... | 5,793 ..... | 45 .....  | 67 .....  | 2,771 ..... | 123 ..... |
| 28 ..... | 5,748 ..... | 50 .....  | 68 .....  | 2,648 ..... | 123 ..... |
| 29 ..... | 5,698 ..... | 56 .....  | 69 .....  | 2,525 ..... | 124 ..... |
| 30 ..... | 5,642 ..... | 57 .....  | 70 .....  | 2,401 ..... | 124 ..... |
| 31 ..... | 5,585 ..... | 57 .....  | 71 .....  | 2,277 ..... | 134 ..... |
| 32 ..... | 5,528 ..... | 56 .....  | 72 .....  | 2,143 ..... | 146 ..... |
| 33 ..... | 5,472 ..... | 55 .....  | 73 .....  | 1,997 ..... | 156 ..... |
| 34 ..... | 5,417 ..... | 55 .....  | 74 .....  | 1,841 ..... | 166 ..... |
| 35 ..... | 5,362 ..... | 55 .....  | 75 .....  | 1,675 ..... | 160 ..... |
| 36 ..... | 5,307 ..... | 56 .....  | 76 .....  | 1,515 ..... | 156 ..... |
| 37 ..... | 5,251 ..... | 57 .....  | 77 .....  | 1,359 ..... | 146 ..... |
| 38 ..... | 5,194 ..... | 58 .....  | 78 .....  | 1,213 ..... | 132 ..... |
| 39 ..... | 5,136 ..... | 61 .....  | 79 .....  | 1,081 ..... | 128 ..... |
| 40 ..... | 5,075 ..... | 66 .....  | 80 .....  | 953 .....   | 116 ..... |
| 41 ..... | 5,009 ..... | 69 .....  | 81 .....  | 837 .....   | 112 ..... |
| 42 ..... | 4,940 ..... | 71 .....  | 82 .....  | 725 .....   | 102 ..... |
| 43 ..... | 4,869 ..... | 71 .....  | 83 .....  | 623 .....   | 94 .....  |
| 44 ..... | 4,798 ..... | 71 .....  | 84 .....  | 529 .....   | 84 .....  |
| 45 ..... | 4,727 ..... | 70 .....  | 85 .....  | 445 .....   | 78 .....  |
| 46 ..... | 4,657 ..... | 69 .....  | 86 .....  | 367 .....   | 71 .....  |
| 47 ..... | 4,518 ..... | 67 .....  | 87 .....  | 296 .....   | 64 .....  |
| 48 ..... | 4,521 ..... | 63 .....  | 88 .....  | 232 .....   | 51 .....  |
| 49 ..... | 4,458 ..... | 61 .....  | 89 .....  | 181 .....   | 39 .....  |
| 50 ..... | 4,397 ..... | 59 .....  | 90 .....  | 142 .....   | 37 .....  |
| 51 ..... | 4,338 ..... | 62 .....  | 91 .....  | 105 .....   | 30 .....  |
| 52 ..... | 4,276 ..... | 65 .....  | 92 .....  | 75 .....    | 21 .....  |
| 53 ..... | 4,211 ..... | 68 .....  | 93 .....  | 54 .....    | 14 .....  |
| 54 ..... | 4,143 ..... | 70 .....  | 94 .....  | 40 .....    | 10 .....  |
| 55 ..... | 4,073 ..... | 73 .....  | 95 .....  | 30 .....    | 7 .....   |
| 56 ..... | 4,000 ..... | 76 .....  | 96 .....  | 23 .....    | 5 .....   |
| 57 ..... | 3,924 ..... | 82 .....  | 97 .....  | 18 .....    | 4 .....   |
| 58 ..... | 3,842 ..... | 93 .....  | 98 .....  | 14 .....    | 3 .....   |
| 59 ..... | 3,749 ..... | 106 ..... | 99 .....  | 11 .....    | 2 .....   |
| 60 ..... | 3,643 ..... | 122 ..... | 100 ..... | 9 .....     | 2 .....   |
| 61 ..... | 3,521 ..... | 126 ..... | 101 ..... | 7 .....     | 2 .....   |
| 62 ..... | 3,395 ..... | 127 ..... | 102 ..... | 5 .....     | 2 .....   |
| 63 ..... | 3,268 ..... | 125 ..... | 103 ..... | 3 .....     | 2 .....   |
| 64 ..... | 3,143 ..... | 125 ..... | 104 ..... | 1 .....     | 1 .....   |
| 65 ..... | 3,018 ..... | 124 ..... |           |             |           |

The meaning of this table will be obvious on inspection. Out of 10,000 infants born alive, it was observed



that 1,539 died before attaining one year of age, leaving 8,461 surviving at age 1. Of these, 682 died during the next year, leaving 7,779 who attained age 2. At age 30 the original 10,000 were reduced to 5,642, and 57 of these died before attaining 31. At age 104 only one person survived, and he died within the next year.

Another form in which Tables of Mortality have been sometimes arranged, is what is called a Table of Decrements, in which the same number of persons are supposed to be alive at every age, and the proportion who die in the next year is set down in the tables. Such a table might be constructed by a simple arithmetical process, from one in the form given above. For instance, a reference to that table shows that out of 5,642 persons aged 30, 57 die in the next year—hence, by proportion, we find that out of 10,000 such persons, 101 will die.

A Table of Mortality is sometimes presented in the form of what is commonly called an Expectation Table, but the facts on which such a table is based must have been first tabulated in the first form given above, and from that table it is found by the following process:—Add together the numbers living at all ages above the given one, divide by the number living at the given age, and add half-a-year to the result. Suppose it is required to find the mean duration of a life aged 100, by the Carlisle Table. In the table given above, we find the sum of the numbers living at 101 and upwards are  $7 + 5 + 3 + 1 = 16$ . The number living at age 100 is 9. Dividing 16 by 9, and adding .5, we obtain 2.278, the mean duration or expectation at age 100 by the Carlisle Table.

In our last Tract (Probabilities) we gave a simple illustration of the application of the Theory of Chances

to find the probability of living a year. It will be obvious that, having a table of the survivors at all ages, a precisely similar process will give the probability of surviving any given number of years, or of dying before attaining a specified age. For instance, we find in the Carlisle Table, that out of 5,009 persons aged 41, 4,000 only will be living at age 56—991 having died in the interval. Hence the probability of a person aged 41 living to attain 56 is about 4.5th, and the probability of his dying in the interval is 1.5th; the sum of these fractions is 1, which, it will be remembered, represents absolute certainty, that certainty being the sum of the probabilities that he will either live or die, one of which events must of course take place.



## VI.

AN ACCOUNT OF SOME OF THE PRINCIPAL TABLES OF  
MORTALITY.

**I**N our last Tract we gave a sketch of the nature of a Mortality Table. We shall now proceed to give a brief account of the principal Tables that have been constructed.

Probably the earliest statement of the statistics of mortality was that prepared by Ulpianus, an ancient Roman. After him we meet with no information on the subject till we come to the 17th century, when John de Wit, Pensionary of Holland, wrote the earliest known treatise on Life Annuities. About the same time Captain Graunt, a citizen of London, made an examination of the numbers and causes of death among the Londoners in the early part of the reign of Charles II. A little later Halley constructed his Tables on the basis of the mortality at Breslau, in Silesia, between 1687 and 1691. About the same time Deparcieux founded a Table on the experience of the French Tontines, and another on the mortality of the monks and nuns in the Paris monasteries and convents. The Chester Table was constructed by Dr. Haygarth, towards the end of the 18th century. All these tables are interesting to the student of the history of vital statistics, but none of them have been practically used for Life Assurance purposes in this country. The earliest table which has been so used—

at least, to any considerable extent—was the celebrated Northampton Table, constructed by Dr. Price on the statistics of the Parish of All Saints, Northampton. The observations extended over 46 years, and comprised 4,220 baptisms (not births) and 4,689 deaths. His data were defective in several particulars, and the final results showed a rate of mortality decidedly too high. The estimation in which this table was held, however, led to its adoption by the “Equitable” Assurance Society, and many others, as well as by the Government for its annuity transactions. The nature of the table, erring as it does in the direction of excessive mortality, though it involved the Government in heavy losses, was a cause of high profit to the insurance companies, and it is probable that this circumstance was one cause of the rapid progress of life insurance business. At least, it may reasonably be conjectured, that had a table erring in the other direction been adopted by the earlier companies, the development of insurance business would have been sensibly retarded.

The Carlisle Table has been more extensively used than any other as the foundation for insurance transactions; and although it is probable that it will before long give place to more modern tables, it is still very largely used, both because, notwithstanding certain defects, it represents with tolerable accuracy the actual mortality among middle-class lives, and because so large a number of monetary tables of every kind have been founded upon it. It was deduced from observations made at Carlisle, between 1780 and 1787, by Dr. Heysham. The table was actually constructed by Mr. Joshua Milne, and exhibits some singular anomalies in various parts, resulting from the small number of deaths observed—only about 1,800. This



table appears in full in the preceding tract, and it will be seen that it extends to age 104—a circumstance which makes it necessary to be cautious in using it for calculating values at advanced ages. For instance, the *Expectation* at 95 appears by the table to be actually greater than at 89—a paradoxical result which did not escape Mr. Milne's observation, and which he ingeniously explains, but which must be carefully kept in view and allowed for in using the table at this point. A similar, but less important, irregularity occurs at the middle of the table; but these anomalies do not affect the usefulness and substantial accuracy of the table for practical purposes.

The Mean Mortality Table, constructed by Mr. T. R. Edmonds, actuary of the Legal and General Assurance Society, published in 1832, was based upon a theory of the author's on the existence of a natural law of mortality. It was thought very highly of by its constructor, but has not been extensively used.

Mr. J. Finlaison's Government Annuity Tables were founded on observations on Government annuitants' lives, beginning with the year 1808. They are more useful for annuity than for assurance transactions.

The English Life Tables, three in number, are based upon mortality and census returns for the whole of England and Wales, embracing observations extending over 17 years, and comprising many million deaths, in these respects contrasting very markedly with the Northampton and Carlisle Tables. Their author is Dr. William Farr.

The table known as the Experience Table was published in 1843, and exhibits the mortality experienced by seventeen of the principal assurance companies, and has been largely used for assurance purposes.

The Institute of Actuaries Table is probably the best that has yet been produced, and it is not likely that it will be superseded for a long time to come. It exhibits the experience, down to 1863, of twenty offices, and is of a very elaborate character. Distinct tables have been formed showing the mortality experienced by the offices among healthy males and healthy females respectively; and another and remarkable table exhibits the mortality among assured lives when the experience of the first five years of assurance is not taken into account. The reason why this last table was formed, was to exclude the effect of selection—for as every life had, of course, been selected after medical examination, this would necessarily affect the rate of mortality, at least for a short time. But as the effect of selection practically disappears after five years, the table in question exhibits the true rate of mortality among assured lives when they have lost their selected character. Mr. Meikle has compiled a valuable table, showing the separate experience of the Scotch offices which contributed to the Institute experience.

Some few offices have also published the results of their individual experience, the principal being the Equitable (two tables), Amicable, Eagle, and Economic.

We may here mention the celebrated hypothesis of De Moivre, which we ought not to omit to notice in connection with a review of Mortality Tables, past and present. Abraham de Moivre was one of the refugees in England consequent upon the revocation of the Edict of Nantes, and devoted himself to mathematical pursuits, principally in connection with the chances at play, and life contingencies. His hypothesis is very simple, and easy to recollect. It is, that out of 86



children born, one will die in each year, until all are extinct. Hence the *Expectation* at any age is found by the simple process of deducting the given age from 86, and halving the result. This hypothesis agrees nearly with the Northampton Table at the middle ages, but it fails at the beginning and end of life. Another and more elaborate hypothesis is that of Mr. Benjamin Gompertz, who thought that the "power to resist destruction" diminishes in a certain fixed ratio as age increases. With high mathematical ability and knowledge, he pursued the investigation of this theory, and arrived at an algebraical expression which gives results nearly approximating to those derived from actual observations.

The following table shows the *Expectation* of life at various ages, as deduced from some of the best known tables of mortality.

*Table of the Expectation of Life for every Ten Years of Age, according to various Mortality Tables.*

| Age. | De Moivre. | Northampton. | Carlisle. | Equitable (Davies). | Edmonds. | Experience. | Institute. Healthy Males. |
|------|------------|--------------|-----------|---------------------|----------|-------------|---------------------------|
| 0    | 43         | 25.2         | 38.7      | —                   | 38.7     | —           | —                         |
| 10   | 38         | 39.8         | 48.8      | 48.8                | 45.2     | 48.4        | 50.3                      |
| 20   | 33         | 33.4         | 41.5      | 41.1                | 38.1     | 41.5        | 42.1                      |
| 30   | 28         | 28.3         | 34.3      | 34.0                | 31.4     | 34.4        | 34.7                      |
| 40   | 23         | 23.1         | 27.6      | 27.4                | 25.0     | 27.3        | 27.4                      |
| 50   | 18         | 18.0         | 21.1      | 20.8                | 18.7     | 20.2        | 20.3                      |
| 60   | 13         | 13.2         | 14.3      | 15.1                | 12.6     | 13.8        | 13.8                      |
| 70   | 8          | 8.6          | 9.2       | 9.8                 | 7.7      | 8.5         | 8.5                       |
| 80   | 3          | 4.8          | 5.5       | 5.4                 | 4.2      | 4.8         | 4.7                       |
| 90   | —          | 2.4          | 3.3       | 2.7                 | 2.1      | 2.1         | 2.4                       |

An inspection of the table will show why the offices which adopted the Northampton Table made large profits; it undoubtedly placed too low a value on human life in this country, at least among that class of

the population who insure their lives. The near accordance of the results in the other tables at the middle ages is a proof of the accuracy with which they have been constructed, and an illustration of the average uniformity which prevails in the duration of life when a large number of persons are placed under observation. At the high ages the lives were comparatively few, and hence the greater disparity in the results.



## VII.

## LIFE ANNUITIES.

A LIFE Annuity is a periodical payment during a term dependent upon a life or lives. The most usual form which such an annuity assumes is a yearly payment during the continuance of a single life. The premium payable on an ordinary whole-life assurance policy is such an annuity. A life annuity may be, like an annuity certain, immediate, temporary, or deferred. An *immediate life annuity* is one which, commencing at once, is to continue so long as the annuitant, or person on whose life the annuity is dependent, continues to live. A *temporary life annuity* is one which is to continue a certain number of years, provided the annuitant lives so long. A *deferred life annuity* is one which is to begin at a certain future date if the annuitant be then alive, and may then continue for a further term of years if the annuitant live so long (in which case it is called a *deferred temporary annuity*), or for the remainder of the annuitant's life. Annuities may depend upon more lives than one:—if payment is to be made so long as *all* the lives continue, in other words if it is to cease when the *first* death takes place, it is called a *joint annuity*:—if to continue as long as *any* of the lives continue, in other words is not to cease until the last of the lives drops, it is called a *joint and survivor annuity*. Or it may be

arranged that the annuity is to begin on the failure of a given life, and then to continue during the existence of other lives, in which case it is a *reversionary annuity*. Other and more complicated forms of annuity are occasionally met with, but the above-named are the principal forms which life annuities assume.

The *present value* of a life annuity is the sum which ought to be invested at a given rate of interest to enable the grantor of the annuity to pay it during the continuance of the life or lives. We shall endeavour to give some idea of the principles on which such values are estimated. The actual method adopted in practice is one, a full explanation of which would be inapplicable to the design of this series of papers. It must be understood, that although the method employed in the example we now give is one which could never be used in practice, on account of the amount of labour it entails, it involves exactly the same principle as that on which the practical method is based.

We will suppose that a person on his 50th birthday is desirous of purchasing an annuity of £1, to be paid to him on every succeeding birthday as long as he lives. What ought he to pay to receive this benefit? It is evident that the present value of an annuity is exactly the sum of the separate values of each of the payments of which it consists. We have, therefore, to find the value of the first, second, third, and each subsequent payment, and adding these values together we obtain the value of the annuity. We have then, first of all, to ascertain the largest number of payments that it is possible will have to be made. We will suppose that the grantor bases his calculations on the Institute Mortality Table for Healthy Males, and Four per Cent. Interest. Turning to the Mortality Table, we find that the highest age given to which any of



the lives attained is 97. On this assumption the last payment which it is *possible* that the annuitant will be alive to receive will be the one due on his 97th birthday. Hence the extreme number of payments which it is possible the grantor can have to pay is 47, and provision must be made for each of the 47 possible payments. Our first step, then, must be to construct a table of the values of £1, payable one year hence, two years hence, and so on up to 47 years hence. If we add the results together, we have the value of an annuity certain for 47 years. (See Tract No. III., Annuities Certain.) But it is highly improbable that the annuitant will live to receive *all* the 47 payments. We must, therefore, reduce the value of each of the payments, as already ascertained, according to the *probability* of the annuitant attaining each successive birthday. We have seen, when discussing probabilities, that the chance of a person of a given age attaining another given age, is to be found by taking from the Mortality Table the number living at the elder age and dividing it by the number living at the present age. If we do this for every age from 51 to 97, we shall obtain a series of fractions; and if we multiply each of the values in the former series by the corresponding values in the present series, we shall obtain a third series of smaller values, the sum of which will give the value of the required life annuity. To sum up, the value of a life annuity is the sum of the values of each payment to the end of life; and the value of each payment is ascertained by multiplying the value of £1 due at the end of each such successive year (at the given rate of interest) by the fraction representing the probability of the annuitant being then alive.

The following illustration, quoted from "De Mor-

gan's Essay on Probabilities" (p. 190), will further elucidate the subject. "If we take a very old life, and suppose that of 10 alive at the present time, there will be left at the end of successive years 7, 5, 3, 0, there are three possible payments of the annuity, and the chances of having to make them are  $\frac{7}{10}$ ,  $\frac{5}{10}$  and  $\frac{3}{10}$ . But the first, if made at all, is not made for a year, and the second and third are not made for two and three years. If, then,  $a$ ,  $b$ , and  $c$  be the present values of £1, to be received at the end of one, two, and three years, the values of the several payments are  $\frac{7}{10}$  of  $a$ ,  $\frac{5}{10}$  of  $b$ , and  $\frac{3}{10}$  of  $c$ , the sum of which is the value of an annuity of £1 on any one life of the kind in question."

We may give one further illustration, as the clear understanding of the nature of an annuity is very necessary to the comprehension of Life Assurance subjects generally. Let it be required to find the value of the *tenth* payment of an annuity of £1 on a life aged 30, at 4 per cent., assuming the rate of mortality shown by the Carlisle Table. The present value of £1, payable certainly 10 years hence at 4 per cent. is .67556; that is, the sum of £.67556, or 13s. 6¼d., improved at 4 per cent. compound interest, would amount to exactly £1 at the end of 10 years. This is the sum, therefore, we should have to reserve and invest if we knew that we should certainly be called upon to pay it when the ten years had expired. But it is not certain that we shall have to pay it, because the annuitant may die in the meantime. Now, suppose that we are called upon to grant such an annuity to each of 5,642 persons, each now aged 30; turning to the Carlisle Table, we find that of 5,642 persons aged 30, only 5,075 persons reach the age of 40. We shall, therefore, be called upon to make a



tenth payment to 5,075 persons—that is, we shall have to pay £5,075. To provide for this we have to set aside and invest 5,075 sums of 13s. 6¼d., or £3,428. 9s. 4d. This, therefore, is the sum that the 5,642 annuitants should among them pay to provide £1 to each survivor of them attaining age 40. Dividing £3,428. 9s. 8d. by 5,642, we obtain 12s. 2d., which is the sum that *each* of the annuitants should pay to provide £1 to himself on his 40th birthday, *if he reaches it*. A similar process applied to each year, from 31 to 104, and a summation of the results, will give the value of the entire annuity.

It will be readily understood that, by summing up the series of yearly values for a few years only from the commencement of the annuity, instead of continuing them to the oldest age shown by the Mortality Table, we shall obtain the value of a *Temporary Life Annuity*. Or, if we begin with any payment beyond the first, and reckon onwards to the end of life, we obtain the value of a *Deferred Annuity*.

Joint annuities on two or more lives are calculated on a similar principle, only we have to consider the probabilities that all the lives in question shall be alive at the end of each successive year.

It will be convenient to note here, that in works on annuities, the present value of a life annuity is usually denoted by the letter  $a$ , with the present age written as a suffix. For instance, if the age be 30, it is written  $a_{30}$ , or if the age be denoted by a letter, as  $x$ , it is written  $a_x$ . If the annuity be on joint lives, now aged, say,  $x$  and  $y$ , it is written  $a_{xy}$ .

The present value of a *Reversionary Annuity* to a person now aged  $x$ , beginning at the death of a person now aged  $y$ , is found by subtracting the present value of an immediate annuity on the joint lives from that on

the life aged  $x$ . For if we *grant* two annuities, one on the joint lives, and one to  $x$ , and *receive* one on the joint lives, it is evidently the same thing as granting the one annuity to  $x$  after the failure of the joint existence. Hence it is properly represented by the expression  $a_x - a_{xy}$ .

If we *grant* one annuity on the life of  $x$ , and another on the life of  $y$ , and *receive* an annuity on the joint lives, a little consideration will show that it is equivalent to granting an immediate annuity, to continue as long as either life remains in existence. This is a *Joint-and-Survivor Annuity*, and its present value is represented by the expression  $a_x + a_y - a_{xy}$ .



## VIII.

## LIFE ASSURANCES.

THE method of determining the sums to be paid as the single and annual premiums to secure a sum payable *at the death* of a given person, is similar in principle to that adopted in determining the value of an annuity *during life*. In computing the value of an annuity, we take into consideration the several probabilities that the annuitant will be living at the end of each successive year to receive his annuity: in like manner, in calculating the premium for an assurance, we have to consider the probability for each successive year that the assured will die in the course of that year, so as to entitle his representatives to receive the sum assured. The Table of Mortality shows us how many, out of a certain number living, die at each age; and on the principles already explained in the article on Probability (Tract No. IV.), the chance of death taking place in any given year is found by dividing the number of deaths in that year by the number now living. The Northampton Table shows, for instance, that out of 5,132 persons aged 20, 72 will die before attaining 21, and of the survivors 75 will die before reaching 22. Hence the probability of death in the first year is  $\frac{72}{5132}$ , and the probability of surviving the first year, but dying in the course of the second, is  $\frac{75}{5132}$ . Hence the rule for finding the single premium for an

assurance may be stated as follows:—Form a series of fractions by dividing the number dying in each year, to the end of the table, by the number living at the present age—these fractions represent the several probabilities of death in the course of each successive year during the course of the longest possible lifetime. Then form another series, consisting of the value of £1 due at the end of one, two, three, and following years. Multiply each term of the first series by the corresponding term of the second; a summation of the results will give the single premium required. Now if this rule is carefully examined, it will be seen that each term has been divided by *the same number*: a little consideration will show that this division may be postponed until the last step, and then performed once for all. We may therefore reduce the process to the following rather simpler form:—Multiply the number dying in the first year by the present value of £1 receivable at the end of one year;\* similarly multiply the number dying in the second year by the present value of £1 receivable at the end of two years; proceed in like manner for every year till the end of the mortality table is reached; sum up the results, and finally divide the amount so obtained by the number living at the present age. If, instead of pursuing this course to the extreme limit of possible lifetime, we close our series at any point on the way, we obtain the premium for a term insurance—that is, for a sum of money to be paid at death, provided that event take place within the given term.

Let it be required to ascertain what single premium should be paid by a person aged 50, to secure the payment of £1 in the event of his death within three

\* Assurances are theoretically supposed to be payable at the *end of the year* in which death occurs.



years, assuming Carlisle Mortality and 3 per cent. interest. Now suppose this person is one of a society of 4,397 persons, each aged 50, and each of whom is desirous of effecting such an assurance. We find, on referring to the Carlisle Table, that out of such a society 59 will die in the first year, 62 in the second, and 65 in the third. The Assurance Company will, therefore, have to pay £59 at the end of one year—the present value of which at 3 per cent. is £57.28,—£62 at the end of two years, of which the present value is £58.441—and £65 at the end of three years, of which the present value is £59.48. The sum of these, £175.21, is therefore the amount which the Assurance Company must have in hand to provide for the payment of the claims; and this sum divided by 4,397, the number of the contributors to the fund, gives £0.399 as the contribution of each person—in other words, the single premium for the term assurance of £1 at age 50 for three years is rather more than ninepence-halfpenny, and consequently for £100 is about £4. A larger rate would be actually charged by an Assurance Company, for reasons to be hereafter explained.

Having ascertained the present value of an annuity on a given life, and the *Single* premium for an assurance on the same life, it is easy to find what *Annual* premium should be paid for the assurance. We have simply to add one year's purchase to the value of the annuity, and to divide the single premium by the amount. In explaining the reason of this, we must first note that annual premiums are payable in advance, the first premium being paid at the moment of effecting the assurance. The values of annuities, as given in the tables, are based on the supposition that the first payment of the annuity is to be made one year after

the annuity is entered upon. Consequently, if an annuity of £1 on a given life, first payment one year hence, be worth, say, £17, an *annuity-due*, as it is called—that is, one of which the first payment is to be made at once—is £18; because the purchaser of such an annuity, for a consideration of £18, receives back £1 at once, and is then evidently in possession of a life annuity of £1, first payment one year hence. This being understood, let us see how the rule given above works in practice. Take the case of an assurance of £100 on a life aged 30—the *Single* premium (Carlisle Mortality 3 per cent.) is £40.129, or £40. 2s. 7d. The value of an annuity of £1, *first payment at once*, on the same life, is £20.556. Now if, instead of paying down £40. 2s. 7d. to the Assurance Office, the assured invests it in the purchase of a *life-annuity-due*, he will receive an annuity of £1.952, or £1. 19s. 0d., and paying this as he receives it, from year to year, to the Assurance Company, he is of course entitled to the same benefit as if he had paid the equivalent single premium. It is to be remarked that this is what is called the theoretical or net premium, and is considerably less than would be charged in practice, as a reference to the prospectus of any Assurance Company will show. The difference is due to what is called the “loading,” the consideration of which we postpone to a future occasion.

In our introductory tract we stated that we should, as far as possible, avoid any algebraical or other abstruse methods of demonstration. Perhaps, however, a statement of the method of finding the value of life annuities and single and annual premiums, in which a few very simple algebraical formulæ are employed, may not be out of place. It is taken, with some modifications, from the standard work of David Jones.



Armonia Sp. - Maria  
Lucinda Maria F.<sup>ca</sup>  
Auto Cantos



16 9

7 1,777

0,5

2,277

Johnson



To find the present value of an annuity payable at the end of every year, during the existence of a single life:—Let  $a_x$  denote the value of an annuity on a life aged  $x$ . Let  $l_x$  denote the number living at age  $x$  according to the Mortality Table. Then  $l_{x+1}$ ,  $l_{x+2}$ , &c., will indicate the numbers living at ages  $x+1$ ,  $x+2$ , &c. Hence  $\frac{l_{x+1}}{l_x}$  will express the probability that a life aged  $x$  will attain age  $x+1$ . From this it will evidently follow, by applying the reasoning of the Tract on Life Annuities, that if  $v$ ,  $v^2$ ,  $v^3$ , be the present value of £1 receivable one, two, three years hence, and so on,

$$\begin{aligned} a_x &= \frac{l_{x+1}}{l_x} v + \frac{l_{x+2}}{l_x} v^2 + \frac{l_{x+3}}{l_x} v^3 + \&c., \&c. \\ &= \frac{l_{x+1}v + l_{x+2}v^2 + l_{x+3}v^3 + \&c., \&c.}{l_x} \end{aligned} \quad (1)$$

The &c., &c., means that the operation is to be continued to the end of the Mortality Table, so that if  $z$  be the last age on the Table, the last term of the numerator will be  $l_z v^{z-x}$ .

If the numerator and denominator of this fraction be each multiplied by  $v^x$ , which will not affect the value of the expression, the formula becomes

$$a_x = \frac{l_{x+1}v^{x+1} + l_{x+2}v^{x+2} + l_{x+3}v^{x+3} + \&c., \&c.}{l_x v^x} \quad (2)$$

and the last term of the numerator will, of course, become  $l_z v^z$ . Formula (1) and formula (2), if worked out for any given age, would give exactly the same result, but the advantage of the latter over the former may be seen by taking as examples the separate ages

of 96 and 95 in the Carlisle Table. By formula (2),

$$\begin{aligned} a_{96} &= \frac{l_{97}v^{97} + l_{98}v^{98} + \dots + l_{104}v^{104}}{l_{96}v^{96}} \\ a_{95} &= \frac{l_{96}v^{96} + l_{97}v^{97} + l_{98}v^{98} + \dots + l_{104}v^{104}}{l_{95}v^{95}} \end{aligned}$$

On comparing the expressions for these two values, we observe that, in finding the value at age 95, every term is introduced which was employed in finding the value at the age of 96; so that it costs very little more trouble to find the value at both the ages than to find the value at one of them only: but had the first expression been used, the operations employed in finding the value at 96 would not have afforded direct assistance in finding the value at age 95.

The single premium for an assurance of £1 payable at the end of the year in which a person now aged  $x$  shall die, may be found by an analogous method. If  $A_x$  be the symbol for such a single premium, and  $d_{x+1}$  the number of persons who, according to the Mortality Table, die between the ages  $x$  and  $x+1$  (or, if  $l_x - l_{x+1} = d_{x+1}$ ), then putting into symbolical form the results of the early part of this paper, it is evident that

$$A_x = \frac{d_{x+1}v + d_{x+2}v^2 + d_{x+3}v^3 + \&c., \&c.}{l_x}$$

Multiplying numerator and denominator by  $v^x$ , as we did in the case of the annuity,

$$A_x = \frac{d_{x+1}v^{x+1} + d_{x+2}v^{x+2} + d_{x+3}v^{x+3} + \&c., \&c.}{l_x v^x}$$

The annual premium being the quotient obtained by the division of the single premium by the value of



an annuity due on the life, if  $P_x$  be the symbol for the annual premium for an assurance of £1 on a life aged  $x$ , then

$$P_x = \frac{A_x}{1 + a_x}.$$

We hope that those of our readers who have studied the earlier numbers of our Tracts, but to whom mathematical formulæ are unfamiliar, will not be discouraged by the perhaps rather formidable appearance of the above expressions. No profound knowledge of mathematics is necessary for their comprehension, the most elementary acquaintance with algebra being all that is required. In fact, they only express in a concise form what we have endeavoured to explain, in this and the preceding paper, in ordinary language and common arithmetic.

## IX.

## PREMIUMS.

THERE is, at first sight, a strong resemblance between Fire Insurance and Life Assurance. In both cases a certain sum is guaranteed on the happening of a certain event, in consideration of an annual payment. In the first case, the sum insured is payable on the destruction of the insured house by fire; in the second case, on the death of the person who is the subject of the assurance. Beyond this, however, the similarity does not extend; the insured house *may* never be burnt, but the assured life *must* terminate. Moreover, the probability that any given house will be destroyed by fire within a given limited time—as, for instance, a year—is, in all ordinary cases, no greater at one period than at another; but the probability that a given life will terminate within a year from any given time is constantly increasing, at least in the case of all persons who have passed the age of childhood. Hence the same annual premium will insure a house against fire for twelve months, year after year; but if life assurances were carried on on the same principle, a constantly increasing premium would be required to meet the constantly increasing risk.

In practice, however, as is well known, an increasing premium is not required, even although the risk is growing greater and greater every day. The reason of



this is that the constant annual premium actually charged for an ordinary life assurance is an average premium—that is to say, it is more than is required in the early years of an assurance for the risk of those years, and in the later years is less than would then be required for a new assurance.

Thus, at age 30, the net premium required to assure £100, to be paid at the end of the year if death occurs within the year, is 15s. (Institute Mortality 3 per cent.) At age 31 the net premium for a similar assurance is 15s. 5d.; at age 32 it is 15s. 9d.; at 33 16s. 1d.; and so on—the premium for each successive year being larger than that for the preceding year, till at age 40 it would be £1, at age 53 £1. 16s. 2d., and at age 60 £2. 17s. 8d., increasing with great rapidity at the older ages. It might be arranged, therefore, that an assurance extending over the whole of life should be paid for by these constantly increasing premiums; but in practice, as is well known, a constant or average premium is charged. At age 30 (on the same data as above) the net annual premium is £1. 17s. 7d.

Now, confining our attention for the present to the first year only of the foregoing series, we observe that 15s. is enough to meet that year's risk—that is to say, if a large number of persons, each aged 30, were to contribute 15s. each, the total amount of their premiums, together with one year's interest, would exactly provide for £100 to be paid to the representatives of each of the persons who died within the year. This will be obvious if we look to the Mortality Table. We there find that of 89,865 persons aged 30, 694 die within the next year. Now if each of these 89,865 persons contribute 15s., the total amount of their contributions is £67,398. 15s. 0d. If to this we add

interest at 3 per cent. (£2,021. 19s. 0d.), we find that the fund at the end of the year has become £69,420. 14s. 0d., or a little more than £100. for the representatives of each of those who have died. The small excess arises from the fact that 15s. is just a little more than the true net premium, which is accurately £74976, or more than 14s. 11¼d., but not quite 15s.

Fifteen shillings, then, from each person would be sufficient to meet the claims of the year. But suppose each contributor had paid £1. 17s. 7d., which is the average whole life net premium, as explained above, what becomes of the difference between this and the 15s.? It is obvious that at the end of the year, *after the death claims have been paid*, a sum of £1. 2s. 7d. multiplied by 89,865, together with one year's interest, or altogether £104,517, remains in hand. This has to be reserved and invested to assist in providing for the years when the average premium will be less than the risk premium; it is required to help to make up the deficiency in the payments of the survivors during later years. Year after year the survivors pay their £1. 17s. 7d. each, of which premium a constantly increasing proportion is required to meet the risk of the year, until at length it is not sufficient for it, and it becomes necessary to draw upon the reserve that has been accumulating.

Were we to pursue the tedious process of tracing the contributions, adding at the end of each year the interest received, and deducting the death-claims, we should find that the premiums would in the end exactly meet the claims, assuming, of course, that the deaths took place precisely as laid down in the Mortality Table, and that the rate of interest on which the premium was calculated was exactly realised on the investment of the funds. We should also find that



the fund, or *reserve*, would gradually increase up to a certain point, and then diminish, until it would ultimately vanish at the end of the year in which the last death took place.

The above illustration supposes that each of the entrants continues to pay his premium so long as he lives. If any of them should discontinue their payments they are entitled to their share of the reserve-fund, inasmuch as by their retirement they relieve the trustees of that fund (the assurance company) from the necessity of making provision for the payment that would otherwise have to be made at their death. Hence a life policy has what is called a *Surrender Value*, which a fire or marine insurance policy has not; that value arising from the fact of a *constant* average premium being paid for a continually *increasing* risk. On the subject of Surrender Values, however, we shall have more to say hereafter.

It will be observed that in our illustration we have frequently used the term *net premium*. Now if the premiums given in this and former papers be compared with those actually charged by assurance companies, it will be seen that the latter are considerably larger. The *net premiums* are the exact sums required to provide certain benefits on certain assumptions—those charged in practice, usually called the *gross premiums*, are arrived at by making additions to the net premiums, to provide against the possible failure to realise the assumed rate of interest, or the possible occurrence of a heavier rate of mortality than that exhibited by the table of mortality on which the net premiums are calculated, and to provide for the necessary expenses of carrying on the business, such as rent, salaries, fees, stamps, and commission to agents, &c. For example, a premium of £1. 17s. 7d., paid annually by a large num-

ber of persons aged 30, would, as we have seen, provide £100 at the death of each of them, *if* the deaths took place exactly in accordance with the "Institute" table of mortality among healthy males, *if* the rate of interest realised were exactly three per cent., and *if* there were no charges for the management of the fund, and no commission paid for the collection of the premiums. But as some charges must be paid, and as it is practically certain that the mortality will actually be at least slightly different from that on which the calculations are founded (and we cannot safely assume that the deviation will be in a favourable direction), and as losses are always possible in the investment of the most carefully managed funds, the net premiums must be increased, or, as it is usually termed, *loaded*, to meet those necessary charges and possible contingencies. Hence the premium actually charged for an ordinary whole-life assurance of £100 on a life aged 30 (without profits) is usually about £2. 5s., or 20 per cent. higher than the net premium as found theoretically on the basis we have assumed in our example.



## ENDOWMENTS, CONTINGENT ASSURANCES, ETC.

A CONTRACT to pay a sum of money to a person on his reaching a certain age, if he reaches it, is termed an *Endowment*. If the payment is to be made on attainment of the given age, *or* at death, should that event take place before it is attained, the contract is known as an *Endowment-Assurance*. The principles involved in determining what single or annual premiums should be charged to secure these benefits, are the same as those involved in determining the premium for an ordinary life assurance, which we have examined in a previous Tract. It will, however, afford a further illustration of those general principles if we look a little closely at their application to this particular class of benefits.

The single premium to secure £100 to a person now aged 30 on his attaining the age of 40 is evidently the present value at the assumed rate of interest, of £100 payable ten years hence, multiplied by the fraction which represents the probability that the life in question will survive ten years.\* Taking the Carlisle Table of Mortality, we find that of 5,642

\* The process of multiplication commonly involves the notion of increase, and where whole numbers are concerned this notion is a just one. The product of the multiplication of two whole numbers is greater than either of them. But it may be as well to remind the reader, that if a whole number is multiplied by a quantity less than unity, that is, by a proper fraction, the product is *less* than the original whole number; and if two proper fractions be multiplied together, the product is less than either of the original fractions.

persons now aged 30, 5,075 will survive to age 40. Hence the probability of reaching that age is, in the case of any one of them  $\frac{5075}{5642}$ , or nearly  $\frac{9}{10}$ . The sum which now invested at 3 per cent. compound interest will exactly reach £100 in ten years is £74. 8s. 2d. Multiply this by the fraction representing the probability, and we obtain £66. 18s. 8d., which is the single premium to secure £100 on the attainment of age 40 by a person now aged 30. That this premium is correct may be proved by showing its sufficiency. Let each of 5,642 persons pay £66. 18s. 8d. (or more strictly £66.931): the total amount of the fund will be £377,650. This, if invested at 3 per cent., will amount at the end of ten years to £507,500, being exactly £100 to each of the 5,075 survivors. Now, in this case, each contributor to the fund *sinks* his premium, and runs the risk of losing it by dying in the meantime: for in the case of a simple Endowment the representatives of persons dying before attaining the specified age have no claim on the fund. But suppose in the above case it were desired, not only to provide £100 for each of those who reached the age of 40, but also £100 to be paid to the representatives of each of the 567 persons dying between 30 and 40. Such a contract, as explained above, is called an Endowment Assurance, and consists evidently of an Endowment combined with a temporary assurance. The single premium for such a benefit, therefore, will be equal to the single premium for the Endowment obtained as above, added to the single premium for an assurance of £100 payable if death take place within ten years. The principles governing the ascertainment of such a premium are explained in Tract No. VIII., and making the calculation we shall find that it amounts to £8. 11s. 2d. Adding this to the single premium for the Endowment, which we have seen is



£66. 18s. 8d., we obtain £75. 9s. 10d. as the single premium for the combined benefit, or Endowment Assurance. In other words, if any large number of persons, each aged 30, were to pay into an Assurance Office the sum of £75. 9s. 10d., the company could pay £100 to each of those who should attain the age of 40, and a like sum to the representatives of each of those who should die in the meantime (such latter payments to be made at the end of the year in which the death took place), provided that the mortality experienced coincided with that exhibited by the Carlisle Table, and that the fund was invested at 3 per cent.; and provided also that no expenses were incurred in the management of the fund. To provide for the necessary expenses, of course an addition or loading would have to be charged over and above the net premium.

The annual premium for the Endowment or Endowment Assurance, or in short for any benefit of the nature of an assurance, is deduced from the single premium by dividing it by the suitable *annuity due*. In a former paper, we showed that in the case of *whole-life* assurance, we calculate the annual premium by dividing the single premium by the *whole-life* annuity due. Similarly, in the case of an Endowment, Endowment Assurance, or Temporary Assurance, we divide the single premium by the *Temporary* Annuity due, inasmuch as the premiums are of course payable only during the currency of the risk, or until the Endowment becomes payable.

Assurances may be effected dependent upon more lives than one, and the essential principles upon which the values of such assurances and the premiums payable in respect of them are ascertained, are the same as in the case of single-life assurances; but inasmuch as we have to take into consideration the more compli-

cated probabilities of joint existence, and in some cases of the order of survivorship, the calculation of such values is considerably more complex than in the case of a single life. We do not purpose entering upon any minute examination of the processes applicable to cases when more than one life is involved, but will content ourselves with a few remarks upon them.

The simplest form of assurance on more than one life is what is known as a Joint-Life Assurance, that is, one in which the amount assured is payable on the death of the *first* of two or more lives; in other words, on the failure of the joint existence. The single premium is obtained by substituting the fraction representing the probability of the failure of joint existence in every year for that of the failure of the single life in the ordinary whole-life assurance formula; otherwise the process is exactly the same. The annual premium is deduced from the single premium, by dividing the latter by the annuity-due on the joint lives.

When the sum assured is to be payable on the occurrence of the death of the last survivor of two or more persons, the contract is usually termed a Longest-life Assurance. When the sum is payable on the occurrence of the death of one of two given persons, provided he dies in the lifetime of the other, the contract is termed a Contingent Survivorship Assurance. The computation of the premium for a contract of this nature is of a complex character. The nature of the contract itself, and the practical use of this kind of assurance, will perhaps be more easily understood by means of an illustration. Suppose A, aged 30, is entitled by a will or settlement to property worth £1000, on the death of B, aged 70, provided he is alive when B's death takes place; but if he should be the first of the pair to die the property is to devolve upon a third person. Now A may very reasonably suppose that he



is, to use a popular expression, pretty certain to come into the property, inasmuch as his age is so much less than that of B. Nevertheless, the uncertainty of life is such that his elderly friend *may* survive him, and so the property may be lost to his family. By effecting a policy, however, the conditions of which are that £1000 are to be paid to his family at his death, supposing he should die first, he converts the contingent benefit into a certainty. It is obvious that the premium, single or annual, will be comparatively small. If annual, it will, of course, be payable during the joint continuance of the two lives; for at the death of either the contract will be at an end. If the joint continuance is determined by the death of A, the office pays the £1000 assured, and the property will pass from A's family—if by the death of B, A obtains possession of the property, and there is no longer any necessity for the assurance; the company retaining the premiums paid, in consideration of the risk it has incurred.

There is another sort of assurance contract sometimes entered into, partaking of the nature of both annuity and assurance, called a Contingent Reversionary Annuity Assurance, that is, an assurance of a life annuity to B, on the death of A, supposing A dies first. This is applicable to the case of a man who desires, in case he should die before his wife, to insure to her a fixed income for the rest of her life. It might be supposed that this would be a popular form of insurance, but as a man whose wife is about the same age as himself is almost as likely to survive her as she is to survive him, the ordinary form of an insurance of a capital sum to his representatives, whoever they may be, is more favoured. The premium, of course, is larger, proportionately, but a certain benefit is generally preferred to one that is contingent.

## XI.

## SURRENDER VALUES.

WE have seen in a former paper that the reason why a life assurance policy possesses a surrender value, is that a continually increasing risk is undertaken by the assuring company in consideration of a uniform annual payment, which, being at first considerably more than sufficient to pay for the risk of the early years, leaves in the hands of the company, after paying its proper share of current claims, a surplus to be reserved and invested at compound interest to meet the high risk of future years, when the premium shall have become too small for that risk. If, therefore, the assured is willing, or desires, to terminate his contract, or to relieve the company from the obligation of paying the sum assured at his death, it is obvious that the surplus fund held by them in anticipation of that event should, in equity, be returned to him. This constitutes, in theory, the surrender value.\*

It would be possible to ascertain exactly what this surplus fund amounts to at any time by a process of calculation based on the principle indicated in the preceding paragraph. Practically, however, the value of a policy is usually estimated in a different way, and based on a different process of reasoning.

The contract, represented by an ordinary life policy, may be regarded as consisting of two obligations. On

\* Strictly, not the whole of the surrender value, but by far the principal part of it.



the one hand, there is the undertaking of the office to pay a specified sum at the death of the assured; on the other hand, there is the undertaking of the assured to pay annually, as long as he lives, a certain premium. In practice, of course, there is no actual undertaking on the part of the assured; but the contract may be conveniently thus regarded for our present purpose. The obligations of the parties being mutual, their values at the outset must be equal to each other; that is, the present value of the premiums payable by the assured must be exactly equal to the present value of the sum payable by the assuring company. But as time goes on, and the assured pays premium after premium, it is evident that he has fewer and fewer premiums to pay, and consequently the value of the future premiums is constantly diminishing. On the other hand, as the period at which the sum assured will become payable is continually approaching, the present value of it from time to time is constantly increasing. The effect of these constant changes of value is, that the value of the difference between the two sets of obligations is constantly increasing. This difference—being, so to speak, the balance of the account between the assurer and assured—is, of course, always in favour of the latter, and is the equitable sum that should be paid for the cancelling of the contract.

Let us take the case of a policy effected by a person aged 30 for £100, at an annual premium of £1'952, being the net premium by the Carlisle Table of Mortality at 3 per cent. interest. We will not embarrass ourselves for the present with the consideration of loaded premiums. Now, the present value of £100, payable at the death of a person aged 30, at the assumed mortality and interest, is £40'129. The value of an *annuity-due* at age 30 is £20'596 for

each £1, and, consequently, the value of the future premiums just before the first premium is paid is  $£19'52 \times 20'556 = 40'129$ . The obligations of the assured and assurer are thus seen to be exactly equal at the beginning of the transaction. Now, let us pass on for ten years, and see how the lapse of time has affected the value of the contract. Ten premiums have been paid, the eleventh is just due. The age of the assured is now 40. The value of £100, payable at the death of a person aged 40 is £47'156, and this is the value of the obligation of the office regarded apart from the obligation of the assured to pay the premiums. The value of this latter engagement is ascertained by multiplying the premium by the value of an annuity due at age 40, or  $1'952 \times 18'143 = 35'415$ . If it be now desired to cancel the contract, the office must pay £47'156, in consequence of its being relieved from its obligation to pay the sum insured, and the assured must pay to the office £35'40, being the present value of the future premiums which the office expected from him, and consequently a balance of £11'741 stands against the office, to be handed to the assured to close the transaction.

If assurance companies could transact business at net premiums—that is, if there were no commission or other expenses connected with them, or other practical considerations to be attended to, the surrender value, as calculated above, would exactly enable a policyholder to transfer from one company to another without loss. In the above case, suppose the policyholder, immediately on receipt of the £11'741, invests it in the purchase of a life annuity-due, which at age 40 would, at 3 per cent. (Carlisle Mortality), produce £6'641 per annum first payment at once. The possession of this annuity enables him to go to another



office, and to effect a new policy, practically at the same rate as before, notwithstanding his increased age. For although the new policy will cost him £2·593 a year, the net premium at age 40, he will only have to pay £1·952 out of pocket, the same as under the old policy, his annuity which he bought with the proceeds of its surrender enabling to make up the difference exactly, since £1·952 + £·641 = 2·593.

The symbol which is generally used in actuarial works to denote the value of a policy is  $V_{x|n}$ , where  $x$  is the age at entry, and  $n$  the number of years elapsed. It may be useful to give here the three principal expressions denoting the value, in an algebraical form.

$$V_{x|n} = A_{x+n} - p_x(1 + a_{x+n}) \quad \dots \quad (1)$$

$$V_{x|n} = (p_{x+n} - p_x)(1 + a_{x+n}) \quad \dots \quad (2)$$

$$V_{x|n} = \frac{a_x - a_{x+n}}{1 + a_x} \quad \dots \quad (3)$$

Here  $A_{x+n}$  represents the value of a reversion, or single premium for the assurance of £1 payable at the death of a person aged  $x+n$  years,  $p$  the annual premium, and  $a$  the value of an annuity. These formulæ are applicable only to whole-life assurance with annual premiums.

The above is a sketch of the principles governing the calculation of surrender values. In practice, however, we have to deal with loaded premiums, and with other disturbing influences, which make the surrender value of practice a rather different thing from the surrender value of theory. One important consideration which tends to lower surrender values is the fact that the contract of Life Assurance is, in one respect, one-sided. The assured has always the option of discontinuing his policy, but the office binds itself to its share of the contract on the sole condition that the premiums be regularly paid. It cannot tender and

compel acceptance of the surrender value. Hence, in the matter of surrenders, a "selection against the office" exists—that is, a man in good health, from a mere caprice, or, it may be, unfounded distrust of the office in which he is insured, will surrender his policy in one company to go into another. But the man whose health is defective, knowing full well that he will never be accepted by any other company, is careful to continue to pay his premiums. For the exercise of this option on the part of the assured, it is only right that he should pay, on the same principle that allowance is made in the case of compulsory sales of land or houses.

It is often supposed by persons surrendering their policies that they have actually *lost* the difference between the premiums they have paid and the sum they receive on surrender. It is difficult to convince them that this is not the case. When it is pointed out that if death had taken place the apparent loss would have been on the other side, the rejoinder is often made that death has not, in their case, taken place, and it is useless basing an argument upon what has *not* occurred. To a person using such an argument it should be explained that, although *he* has not died, others assured in the same office as himself have done so, and that a part of his premiums have been used to pay their policies, just as a part of *their* premiums would have been required to pay *his* policy if he had died. It is strange that so much ignorance should prevail on the subject, but every assurance official must have met with cases in which policyholders have expressed their opinion that the whole of the premiums paid by them ought to be returned on demand, and have considered themselves generous in waiving a claim for compound interest. How the death claims would ever be paid, to say nothing of the expenses of the office, if the whole of



the premiums had to be held in reserve against a possible simultaneous demand of all the policyholders for their return in full, has never occurred to them.

These explanations will show the serious fallacy contained in an observation commonly made in connection with the two great failures of the Albert and European Companies. It is frequently remarked that Mr. A., after having paid premiums for so many years, has, by the failure of the company "lost them all." Now, the loss, instead of being *all* his payments, has been really but a fraction of them, probably between one-third and one-half. The rest has been consumed in meeting his share of the death claims and of the expenses of the company. Suppose that Mr. A. paid during ten years a premium of £10 on his life policy in the Albert, and a similar premium of £10 on a fire policy in some other company. The Albert having failed, he tells his friends he has lost £100 by them; but he never supposes that the £100 he paid to the fire company has been *lost*. He knows he has had his money's worth in security, and although no fire has occurred on his premises, and he has never received a penny from the fire insurance company, he nevertheless regards the expenditure as a good investment. He should look upon his life premium in a similar light; but whereas, in the case of the fire policy, the premiums were exactly adjusted to a constant risk, the life premium was calculated on a different principle, as explained in this and a former paper, and was larger than was required for the risk actually incurred. An accumulation of the surplus, therefore, after paying for risk and expenses, ought to have been in the coffers of the company. This, the so-called "value of the policy," *and this only*, is the loss which, in the case supposed, has been sustained.

## XII.

## BONUS.

IF the business of a life assurance company could be carried on without expense, and if its mortality experience and the rate of interest realised on its investments corresponded exactly with the mortality and interest assumed in the computation of the net premiums, then these net premiums paid by the policyholders would be exactly sufficient, in the long run, to meet the engagements of the company. If it were at any time to cease to issue new policies, the funds in hand would soon begin to diminish, and when the last surviving policy-holder died, it would be found that there was remaining in the hands of the company just enough to meet the claim on his policy,—no more and no less. The former papers of this series, taken together, are an attempt to demonstrate that this would necessarily be the case if the practical conditions ever corresponded with pure theory. This, however, can never be. Fluctuations of various kinds occur, sometimes in one direction and sometimes in another. The average duration of the assured lives in a given society may be longer or shorter than was anticipated; the rate of interest realised varies from time to time; losses take place on investments, or perhaps unforeseen profits arise in connection with them. That company would be singularly unfortunate, or managed with strange carelessness, in which these fluctuations should



be always or chiefly in an unfavourable direction. Safety, however, is the paramount necessity of an assurance society; and as the balance of fluctuations may possibly be against it, it is necessary to provide against this contingency, and to "load" the net premiums not only with sufficient to meet the reasonable expenses of management, but with something additional to provide against possible losses.

Good and economical management and favourable mortality experience will probably, however, result in the balance of fluctuation being on the right side, and consequently the premiums paid may prove to have been too much. An equitable distribution of the excess has to be made among those entitled to it, varying according to the constitution of the company or society, and the proportion of the excess so distributed among policy-holders is generally called *bonus*.

If the loading of the premiums has been very carefully and narrowly estimated, so that only moderate expenses of management, and no more than the highest *probable* fluctuation in the rate of interest and mortality are provided for, of course but little surplus is likely to arise, even after several years' operations, and consequently the bonus, if any, will be very small. But as a bonus is very attractive, and is found to have no small influence on the new business of an office, almost every company adopts the expedient of charging those of its assurers who are to participate in profits such a premium as shall ensure a surplus from time to time. The net premiums are accordingly loaded considerably in excess of what is required for expenses and inevitable contingencies, and the excess so paid by the policy-holders is returned to them at intervals—generally every five or seven years. At the end of every such interval, a calcu-

lation is made of the amount which it is necessary to keep in hand to meet the risk under the policies—in other words, an estimate is made of the value of the policies. The actual processes by which this estimate is made are very various in their details, and considerable difference of opinion prevails among actuaries as to the best method of arriving at it. In effect, however, the *present value* of the sums assured being ascertained, the *present value* of the future premiums is deducted from them, and the difference (after making some provision for future expenses) is ascertained. This difference is what is known as the *reserve*, and must be retained in hand to be improved at interest. The difference, again, between the *reserve* and the *assets*, or actual property of the company, is what is called the *surplus*, and this it is which is distributable, either in whole or in part, as bonus. The following example, necessarily somewhat rough, will make the foregoing rather clearer. Suppose a company, which, on investigating its affairs, finds that the total amount of the sums assured by its policies is £2,250,000, and that the total annual premiums payable in respect of these policies amount to £70,000. On carefully computing, with due regard to the ages of the various policy-holders, the sum which it would be necessary to invest at a given rate of interest to provide for the payment of each policy as it falls in, *supposing no further premium were to be paid*, it is found to be, say, £1,050,000. This may be called the *gross liability*. Against this is to be set the present value of the premiums. Now, we must not consider the present value of the whole £70,000 as an asset, because a part of it will be consumed each year as expenses and reserve for bonus. We accordingly strike off, say, £20,000 a year for these purposes, and credit our account with the pre-



sent value of £50,000 a year, which we will suppose is worth, in the aggregate, thirteen years' purchase, or £650,000. This has to be deducted from the gross liability of £1,050,000, and leaves a *net* liability of £400,000. Now, further, suppose that the invested assets of the company—that is to say, its cash, securities, real property, mortgages, &c.—are found to be worth £480,000, it appears that a surplus exists of £80,000, which may be distributed among the various policy-holders as bonus. Briefly stated the account stands thus:—

|                                                                                                                                                  |            |
|--------------------------------------------------------------------------------------------------------------------------------------------------|------------|
| DR.—To present value of £2,250,000 assured                                                                                                       | £1,050,000 |
| „ Surplus .....                                                                                                                                  | 80,000     |
|                                                                                                                                                  | <hr/>      |
|                                                                                                                                                  | £1,130,000 |
| CR.—By present value of £50,000 a year;<br>being annual premium income, after<br>deducting £20,000 a-year for future<br>expenses and bonus ..... | 650,000    |
| „ Invested Assets .....                                                                                                                          | 480,000    |
|                                                                                                                                                  | <hr/>      |
|                                                                                                                                                  | £1,130,000 |

The next consideration, after the divisible surplus has been ascertained, is how it is to be distributed among the assured. Here, again, is a wide diversity of opinion and practice. Theoretically, of course, each policy-holder ought to have returned to him precisely the amount he has paid in excess of his share of the risk and expenses, but the accurate ascertainment of this sum is a matter of considerable difficulty. Practically, methods are adopted which, without being absolutely accurate, are sufficiently near to the truth to do more than substantial justice. We do not purpose entering upon any examination of the merits of the various methods, but will simply give a statement of the forms which *bonus* usually assumes.

The simplest way in which a bonus can be given is by distributing the ascertained surplus in actual cash among the policy-holders. This may be given either in proportion to the premiums paid since the last distribution, or since the first issue of the policy. Sometimes the apportionment is regulated by the value of the policy, and other methods have their various advocates. The results, however, of these various systems will vary but slightly in individual cases, so that the most obvious or plainest system is perhaps practically the best. Cash bonuses are naturally very popular, for to the unreflecting they have the appearance of a gift for which no consideration has been given. How far a cash bonus, however, is a real advantage, may be simply estimated in any given case by the following method. Take the difference between the participating and non-participating rates, and see how much the annual excess has amounted to at compound interest. If the declared cash bonus is less than this amount, the advantage of assuring with profits has been so far less than none. For example, suppose a policy is effected at age 30 for £1000, at a premium of £25. 15s.—a rate which would usually entitle to participation in profits; and further, suppose that a non-participating policy could be effected on the same life in the same office for £1000, at a premium of £22. 5s. At the end of five years, at 4 per cent., the £2. 10s. annual excess will have amounted to more than £14, so that if the cash bonus is less than this, it has been a positive loss to have assured “with profits.” But even if it be a little more, the gain is only apparent, as the chance of death during the five years must be taken into account, in which case no bonus would have been paid, and the difference between the two rates of premium wholly lost.



A *reversionary* bonus is an addition to the sum assured, and is payable only when death takes place, or in the case of endowment-assurances, when the age is reached at which the policy is made payable. Other things being equal, it varies, of course, with the age of the assured at the time of distribution. For example, if a *cash bonus* of £10 has been declared on a policy on a life aged 25, and the same amount of *cash bonus* on another policy on a life aged 75, the corresponding *reversionary bonuses*, or additions to the sums assured which could be taken *instead* of the cash, would probably be about £30 and £12 respectively.

Bonus may also be applied in the permanent or temporary reduction of the premium, and it is very important that the distinction between these two forms should be fully recognised and understood. It is the practice in some companies to declare a reduction in the premium for the next five or seven years only, at the termination of which the premium resumes its former amount, subject, however, to any new reduction that may be made for the next period. Usually the new reduction is as large as or larger than the former, but the disadvantage of the temporary reduction system is that if, by any misfortune, the profits of any period have been materially less than that of preceding periods, it may result in a *reduction upwards*, so to speak, or an actual increase in the premium in comparison with what it has been during the five or seven years preceding. The permanent reduction is, of course, always much smaller than the corresponding temporary reduction, but has the advantage of not being subject to fluctuation; and, as the operation of successive reductions is cumulative, they may ultimately result in the actual permanent extinguishing of the premium.

## XIII.

## CLAIMS.

EVERY active Life Assurance Agent can testify to the ease with which objections can be found to Life Assurance, and to the difficulty there is in meeting them efficiently. Some of them, fanciful and sentimental, are almost proof against logic and reasoning, and must be dealt with by tact and ingenuity. Others, more substantial, may be answered by imparting technical and business information, and thus removing the ignorance from which such objections usually arise. Of this class is the often-urged objection, that the realising of the assured amount, after the death of the policy-holder, is a matter of uncertainty, or, at any rate, of considerable difficulty. The agent is constantly hearing stories of the trouble which Mr. A's widow, or Mr. B's son, had in extracting the amount of the policy from the Assurance Company. A careful examination of the facts will, however, usually result in the discovery that the difficulties have been much exaggerated, and that such as there were, were the consequences of ignorance or carelessness on the part of either the deceased or his representatives. The requirements of assurance companies in proof of death and title are few and simple; and, with a little care and attention, no difficulty or delay ought to take place.



The process of establishing a claim is one with which every agent should make himself familiar, both as regards the formal requirements of the particular office which he represents, and also those general principles which are applicable to all offices. By this means, he will be able readily to dispose of the objections met with in canvassing, to answer many questions frequently put by policy-holders on the subject of wills and assignments, and to instruct claimants as to the course to adopt in obtaining payment of their policies.

An ordinary life policy granted to any person on his own life usually provides that, in consideration of the payment, during life, of a certain annual or other premium, the sum assured shall be paid to the executors, administrators, or assigns, of the assured, at a certain interval (usually three months) after his death shall have been proved to the reasonable satisfaction of the directors of the Life Assurance Company granting the policy. On death taking place it is therefore necessary to prove—

1. That the assured person is dead ;
2. That the claimant is the proper person to receive the amount of the policy, either as his executor or administrator, or as owner of the policy by purchase, as assignee in trust, or as mortgagee with power to give receipt, or otherwise.

First as to proof of death. The claimant should apply to the office for the official forms on which to make out his claim. He will generally be required to furnish a copy of the Registrar's certificate of death, which may be obtained from the District Registrar for a small fee ; a certificate of burial ; a certificate from the medical attendant, certifying the cause of death

and duration of the fatal illness ; and some proof that the deceased was, at the time of making the assurance, of the age stated on the proposal. He should also send in the last premium receipt, in proof that the policy was in force at the time of death. With regard to the proof of age, most offices will admit the age at any time during the currency of the policy on production of satisfactory evidence, such as a copy of the Registrar's certificate of birth, or of an entry in a family Bible authenticated by a statutory declaration. Agents should urge all policy-holders to see to this matter as soon as the policy is delivered to them, and they will by so doing save their representatives one item of trouble in substantiating their claim. The actual proof of the fact of death is generally complete when the above-named documents have been sent in, and a letter stating that the directors are satisfied with the evidence is generally sent to the claimant, stating when and where he can receive the amount assured, subject to his satisfying the company in the meantime that he is the proper person to receive the money and give the receipt.

Then, as to proof of title :—the amount being payable to “executors, administrators, or assigns,” the claimant must prove that he is one of these by lodging with the company either the probate of the will, letters of administration, or deed of assignment. If the deceased has left a will, not having parted with his property in the policy during his lifetime by gift, sale, or mortgage, the process of establishing the claim is very simple. It is the duty of every man possessed of even the smallest amount of property to make a will. By so doing he saves the time, trouble, and money of those he leaves behind him. It is surprising how many there are who die, as it is called, *intestate*, that



is, without having made a will. They have had, perhaps, a superstitious feeling that so doing would, in some inexplicable way, hasten their death; or have had some hesitation as to how they should distribute their property; or, it may be, they have had erroneous notions of the difficulty of making a will, and so the duty has been put off from year to year till death has overtaken them, and their representatives have found themselves involved in the extra trouble and expense of distributing the estate of an intestate. The process of making a will is, in truth, a very simple one. When the property is large, and is to be apportioned among many persons, and especially when there is landed estate, it is in the highest degree unwise to attempt to make a will without professional assistance. But when the property is small, and is to be left to one person absolutely, nothing more is necessary than to state the fact in plain language, to appoint an executor, and to see that the execution or signature of the will is duly made. Every will must be signed by the testator in the presence of two persons, not legatees, who must be present at the same time, who must actually see the testator and each other sign, and the fact that they have done so must be stated at the foot of the will. Detailed information on these points is to be found in popular treatises on legal matters, but even in the most simple case the assistance of a solicitor is cheaply purchased at a guinea, and the satisfaction of knowing that the will is one which will be effectual is in itself worth even a larger outlay. When death takes place the person named as executor will have to obtain *Probate*. In every large town there is an office where probates are granted, and the officials are bound to assist executors in the simple process of conforming to the regulations. In addition

to the stamp the costs are inconsiderable. The will has to be left at the Probate Office, and in the course of a few days an official copy of the will, with the probate annexed, sealed with the seal of the Probate Court, is ready for the executor. This document authorises him to act on the directions in the will, to realise the estate of the deceased, to collect his debts and all money due to him, including, of course, the life policy, and to give receipts effectually discharging the debtors. Of course, he is likewise charged with the duty of paying the debts and obligations of the deceased, and this he must do before he begins to distribute the legacies. Here, again, if the will be long and complicated, or if the estate be insufficient to pay the debts and legacies in full, the executor should consult a solicitor, but in simple cases there is no great danger in dispensing with professional aid. When the deceased has not left a will, his nearest relative should apply to the Probate Office for directions how to proceed. The expenses are larger than where a will exists, and the effects must be distributed in a manner prescribed by Act of Parliament. Should the nearest relative object to take upon himself the trouble and responsibility of acting as administrator, a more remote relative, or even a creditor of the deceased, may obtain *Letters of Administration*. This document is similar in effect to the probate of will, and empowers the administrator to collect the sum assured under a life policy.

In the case of a policy acquired by purchase, or lodged as a security for debt, the owner or creditor ought, to make himself secure, to give notice of the fact to the insurance office *immediately on coming into possession of the policy*. When the person whose life is assured is dead, he should, as soon as the fact of the death is admitted, exhibit the policy and deed of as-



signment at the office of the company. If, on examination, it is found that the deed is one which fully empowers him to give a receipt, he will be paid in due time on so doing. If there is any irregularity in the deed, he will generally be required to obtain the concurrence of the executor or administrator. He may find some difficulty in doing this; and if so, it is very desirable that he should do all in his power to effect an arrangement, as the assurance offices will not enter into the merits of any quarrels between rival claimants; and if either party put any pressure upon them, can relieve themselves by paying the money into the Court of Chancery, and leave the parties to adjust their difficulties before that tribunal, to the certain diminution of the fund in dispute to a greater or less extent.

The executor, administrator, or other claimant, having thus established his claim, has nothing more to do than to attend at the office on the day named, deliver up his policy with the receipt endorsed upon it, and receive his cheque. He ought then to proceed to the office of the agent, express his satisfaction, and fill up a proposal for a policy on his own life, and if he is an executor who has to pay legacies, he should strongly urge every legatee to go and do likewise.

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